Week 4 Worksheet Tuesday

Di Fang

Instructions. Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Today's Topic:
Limit Computing (mainly focus on "∞" type) "$\frac{\infty}{\infty}$" "$\infty - \infty$"

Please inform your TA, if you find any error in the solutions! :)

Find the following limit, indicating $\pm\infty$ when applicable.

1. $\lim_{x\to -1} \frac{t^4}{(t+1)^4}$

$= \frac{t^4}{(t+1)^4}$

2. $\lim_{x\to 0} \frac{|x|}{x}$

DNE

$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = -1$

$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = 1$

must master!

3. $\lim_{x\to -\infty} \frac{-x^{2016}-9x^{-1}+27}{x^{2015}+2x^{2016}-221}$

$= \lim_{x\to -\infty} \frac{(-x^{2016} - 9x^{-1}+27)\frac{1}{x^{2016}}}{(x^{2015}+2x^{2016}-221)\frac{1}{x^{2016}}}$

$= \lim_{x\to -\infty} \frac{-1 - \frac{9}{x^{2017}} + \frac{27}{x^{2016}}}{\frac{1}{x} + 2 - \frac{221}{x^{2016}}}$

$= -\frac{1}{2}$

4. $\lim_{x\to 1^-} \frac{4x^4+10x^3+1}{(x-1)^2}$ *(from 2015 Spring exam)*

DNE

When $x\to 1^-$ $4x^4+10x^3+1 > 0$

$(x-1)^2 > 0$

$\Rightarrow ? = +\infty$

5. $\lim_{x\to +\infty} \frac{\sqrt{9x^4+3x}}{(2x+1)^2}$

$= \lim_{x\to +\infty} \frac{\sqrt{9x^4+3x}\cdot \frac{1}{x^2}}{(2x+1)^2 \cdot \frac{1}{x^2}}$

$= \lim_{x\to +\infty} \frac{\sqrt{9x^4+3x^2}\cdot \frac{1}{\sqrt{x^4}}}{(2x+1)^2 \cdot (\frac{1}{x})^2}$

$= \lim_{x\to +\infty} \frac{\sqrt{9+\frac{3}{x^2}}}{(2+\frac{1}{x})^2}$

$= \frac{\sqrt{9}}{2^2} = \frac{3}{4}$

6. $\lim_{x\to +\infty} \frac{x^2+3\sqrt{x^2+1}}{\sqrt{4x^2-5}}$ *(from 2014 Fall exam)*

$= \lim_{x\to +\infty} \frac{(x^2+3\sqrt{x^2+1})\cdot \frac{1}{x^2}}{\sqrt{4x^2-5}\cdot \frac{1}{\sqrt{x^4}}}$

$= \lim_{x\to +\infty} \frac{1+\frac{3\sqrt{x^2+1}}{x^2}}{\sqrt{\frac{4}{x^2}-\frac{5}{x^4}}}$

$\left(\frac{3\sqrt{x^2+1}}{x^2} = \frac{3\sqrt{x^2+1}}{\sqrt{x^4}} = 3\sqrt{\frac{1}{x^2}+\frac{1}{x^4}}\right)$

$= \lim_{x\to +\infty} \frac{1+3\sqrt{\frac{1}{x^2}+\frac{1}{x^4}}}{\sqrt{\frac{4}{x^2}-\frac{5}{x^4}}}$

$= DNE.$ Since both numerator and denominator are positive, $? = +\infty$.

must master!

7. $\lim_{x\to+\infty} \sqrt{4x^2-3x}-2x$ *(2014Fall)*

$$= \lim_{x\to+\infty} \frac{(\sqrt{4x^2-3x}-2x)(\sqrt{4x^2-3x}+2x)}{\sqrt{4x^2-3x}+2x}$$

$$= \lim_{x\to+\infty} \frac{-3x}{\sqrt{4x^2-3x}+2x} \cdot \frac{1/x}{1/x}$$

$$= \lim_{x\to+\infty} \frac{-3}{\sqrt{4-\frac{3}{x}}+2} = \frac{-3}{\sqrt{4}+2} = -\frac{3}{4}$$

8. $\lim_{x\to+\infty} \sqrt{x^2-3}-x$ *(from 2016 spring)*

$$= \lim_{x\to+\infty} \frac{-3}{\sqrt{x^2-3}+x}$$

$$= 0$$

9. $\lim_{x\to+\infty} \sqrt{x}-3$

$$= +\infty$$

10. $\lim_{x\to+\infty} \sqrt{x^2+x+1}-(x+\frac{1}{2})$ *(from 2015 Fall exam)*

$$= \lim_{x\to+\infty} \frac{x^2+x+1-(x+\frac{1}{2})^2}{\sqrt{x^2+x+1}+(x+\frac{1}{2})}$$

$(x+\frac{1}{2})^2 = x^2+x+\frac{1}{4}$

$$= \lim_{x\to+\infty} \frac{3/4}{\sqrt{x^2+x+1}+(x+\frac{1}{2})}$$

$$= 0$$

11. $\lim_{x\to+\infty} \sqrt{x}-\sqrt{x+3}$ *(From 2015 Spring exam)*

$$= \lim_{x\to+\infty} \frac{x-(x+3)}{\sqrt{x}+\sqrt{x+3}}$$

$$= \lim_{x\to+\infty} \frac{-3}{\sqrt{x}+\sqrt{x+3}}$$

$$= 0$$

The point here is since $x<0$

$x \neq \sqrt{x^2}$ (x: negative; $\sqrt{x^2}$: positive)

$x = -\sqrt{x^2}$

12. [Optional] $\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{x-1}$

Ok to ignore this problem.

$$= \lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{x-1} \cdot \frac{1/x}{1/x}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{x-1} \cdot \frac{-\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{2+\frac{1}{x^2}}}{1-\frac{1}{x}}$$

$$= -\sqrt{2}.$$